

Quasiparticles as composite objects in the RVB superconductor

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We study the nature of the superconducting state, the origin of d -wave pairing, and elementary excitations of a resonating valence bond (RVB) superconductor. We show that the phase string formulation of the $t - J$ model leads to confinement of bare spinon and holon excitations in the superconducting state, though the vacuum is described by the RVB state. Nodal quasiparticles are obtained as composite excitations of spinon and holon excitations. The d -wave pairing symmetry is shown to arise from short range antiferromagnetic correlations.

I. INTRODUCTION

This paper concerns the nature of the superconducting state and the excitation spectrum obtained from the bosonic resonating valence bond (b-RVB) theory of the $t - J$ model. The $t - J$ model, arguably the simplest description of a doped Mott insulator [1], is given by

$$H = -t \sum_{\langle ij \rangle \sigma} c_{i\sigma}^\dagger (1 - n_{i-\sigma}) c_{j\sigma} (1 - n_{j-\sigma}) + J \sum_{\langle ij \rangle} \left[\mathbf{S}_i \cdot \mathbf{S}_j - \frac{\mathbf{n}_i \mathbf{n}_j}{4} \right]. \quad (1)$$

The b-RVB theory of the Hamiltonian (1) yields a consistent description of the doped antiferromagnetic Mott insulator [2]. At half filling, the theory reduces to the Schwinger boson representation of the Heisenberg model [3–5]. At small doping concentrations, the theory describes a cluster spin glass phase, which eventually makes way to a superconducting phase [6]. The theory also exhibits a spontaneous vortex phase where the holons are (Bose) condensed and the spinons are perceived as half vortices by the holon field [7]. Recently, we considered a Ginzburg-Landau formulation of this theory and showed how $\frac{hc}{2e}$ quantization arises in the superconducting state. A spinon, perceived as a half vortex by the holon field, is trapped within a vortex core, thereby leading to a $2e$ flux quantum [8]. In this paper, we present a microscopic description of the superconducting state at zero temperature. In particular, we show explicitly that when the holons are condensed, it costs a logarithmically divergent energy to create a bare spinon excitation. Therefore, a bare spinon excitation is confined. We show that energy considerations only permit neutral $S = 1$ and quasiparticle excitations. We also demonstrate that the short range antiferromagnetic (AF) correlations lead to d -wave pairing symmetry. The results presented in this paper illustrate how a nodal quasiparticle arises from an RVB vacuum, at low energies. This paper complements the macroscopic description of the superconducting state, presented in our earlier work [8].

The paper is organized as follows. For completeness, we devote the next section to a brief summary of the b-RVB formulation of the $t - J$ model (1). This leads to an effective Hamiltonian for the spin and charge degrees of freedom which can be solved within a mean field approximation. The basic results used repeatedly in this paper are summarized. In the following section, we discuss the nature of the superconducting state, and show how d -wave pairing arises in this theory. The excitation spectrum is studied in Section IV. We show that bare spinon (or holon) excitations are confined in the superconducting state, and that only neutral $S = 1$ and composite excitations (that carry both spin and charge) are allowed in the superconducting state. The latter would be the RVB analog of quasiparticles in a BCS superconductor. In the final section, we summarize our results and discuss them in the context of the complete phase diagram obtained within the b-RVB theory.

II. BOSONIC FORMULATION OF THE $T - J$ MODEL

The b-RVB formulation can be regarded as a bosonization scheme for the $t - J$ model (1). The electron operator, $c_{i\sigma}$ is written as

$$c_{i\sigma} = h_i^\dagger b_{i\sigma} e^{i\hat{\Theta}_{i\sigma}} \quad (2)$$

where the holon $h_{i\sigma}^\dagger$ and spinon $b_{i\sigma}$ operators represent bosonic fields. The holon and spinon fields satisfy the constraint, $h_i^\dagger h_i + \sum_\sigma b_{i\sigma}^\dagger b_{i\sigma} = 1$, as in the fermion or d -RVB theories [9]. However, the presence of the nontrivial

phase string operator, $\hat{\Theta}_{i\sigma}$, in (2), makes this theory very different from the d -RVB theories. The phase string operator leads to nonlocal correlations between the charge and spin degrees of freedom on account of hole motion in a background with short range AF correlations. The fermion statistics of $c_{i\sigma}$ is also realized through the phase field

$$e^{i\hat{\Theta}_{i\sigma}} = (\sigma)^{\hat{N}_h} (-\sigma)^i e^{i\Theta_{i\sigma}^{\text{string}}} . \quad (3)$$

The Klein factor $(\sigma)^{\hat{N}_h}$ guarantees anticommutation relations between electron operators with different spins. Unless otherwise stated, this factor will be omitted as it does not play any role in the spin, transport, and single particle channels. The phase string operator $\Theta_{i\sigma}^{\text{string}}$, is a nonlocal operator given by

$$\Theta_{i\sigma}^{\text{string}} \equiv \frac{1}{2} [\Phi_i^b - \sigma \Phi_i^h] , \quad (4)$$

where

$$\Phi_i^b = \sum_{l \neq i} \theta_i(l) \left(\sum_{\alpha} \alpha n_{l\alpha}^b - 1 \right) , \quad (5)$$

and

$$\Phi_i^h = \sum_{l \neq i} \theta_i(l) n_l^h . \quad (6)$$

Here, $n_{l\alpha}^b$ and n_l^h are spinon and holon number operators, respectively. The quantity $\theta_i(l)$ is defined as $\theta_i(l) \equiv \text{Im} \ln (z_i - z_l)$, where $z_l = x_l + iy_l$ is the complex coordinate of a lattice site l .

Using the decomposition (2), we find that the hopping term H_t in (1) can be written as

$$H_t = -t \sum_{\langle ij \rangle \sigma} \left(e^{iA_{ij}^s - i\phi_{ij}^0} \right) h_i^\dagger h_j \left(e^{i\sigma A_{ji}^h} \right) b_{j\sigma}^\dagger b_{i\sigma} + h.c. , \quad (7)$$

where

$$A_{ij}^s = \frac{1}{2} \sum_{l \neq i,j} [\theta_i(l) - \theta_j(l)] \left(\sum_{\sigma} \sigma n_{l\sigma}^b \right) , \quad (8)$$

and

$$\phi_{ij}^0 = \frac{1}{2} \sum_{l \neq i,j} [\theta_i(l) - \theta_j(l)] . \quad (9)$$

The lattice gauge field A_{ij}^s describes fictitious fluxoids bound to spinons satisfying

$$\sum_c A_{ij}^s = \pm \pi \sum_{l \in c} [n_{l\uparrow}^b - n_{l\downarrow}^b] , \quad (10)$$

for an arbitrary closed path c (assuming no spinons along c). The sign \pm on the right hand side of the above equation refers to the clockwise or counter clockwise choices for the path c . The quantity ϕ_{ij}^0 describes a uniform flux threading through the 2D plane with a strength π per plaquette, $\sum_{\square} \phi_{ij}^0 = \pm \pi$. The gauge field A_{ij}^h is given by holons,

$$A_{ij}^h = \frac{1}{2} \sum_{l \neq i,j} [\theta_i(l) - \theta_j(l)] n_l^h , \quad (11)$$

and describes fictitious fluxoids bound to holons,

$$\sum_c A_{ij}^h = \pm \sum_{l \in c} n_l^h . \quad (12)$$

The superexchange term H_J in (1) is given by

$$H_J = -\frac{J}{2} \sum_{\langle ij \rangle \sigma \sigma'} \left(e^{i\sigma A_{ij}^h} \right) b_{i\sigma}^\dagger b_{j-\sigma}^\dagger \left(e^{i\sigma' A_{ji}^h} \right) b_{j-\sigma'} b_{i\sigma'} , \quad (13)$$

From equations (7) and (13), we see that the b-RVB theory can be thought of as a π -flux theory where the π fluxoids are bound to the constituent particles, *i.e.*, the spinons and the holons.

A. Mean field solution

The Hamiltonians for the charge and spin degrees of freedom, (7) and (13) respectively, can be solved within a generalized mean field approximation. We expect the presence of the nonlocal fields A_{ij}^s and A_{ij}^h to cause residual effects of the short range AF correlations within such a mean field treatment. We summarize below, a few results that will be used in this paper. We refer the reader to an earlier paper by one of us for details [2].

A mean field decoupling leads to an effective Hamiltonian, $H_{\text{eff}} = H_h + H_s$, where the effective holon Hamiltonian

$$H_h = -t_h \sum_{\langle ij \rangle} \left(e^{i[A_{ij}^s - \phi_{ij}^0]} \right) h_i^\dagger h_j + h.c. , \quad (14)$$

with $t_h \sim t$. The effective spinon Hamiltonian is given by

$$H_s = -J_s \sum_{\langle ij \rangle \sigma} \left(e^{i\sigma A_{ij}^h} \right) b_{i\sigma}^\dagger b_{j-\sigma}^\dagger + h.c. + \lambda \left(\sum_{i\sigma} b_{i\sigma}^\dagger b_{i\sigma} - (1 - \delta)N \right) , \quad (15)$$

where J_s and J are related by the generalized RVB order parameter Δ^s as $J_s = \frac{\Delta^s}{2}J$. In the above expression,

$$\Delta^s \equiv \Delta_{ij}^s = \sum_{\sigma} \left\langle e^{-i\sigma A_{ij}^h} b_{i\sigma} b_{j-\sigma} \right\rangle . \quad (16)$$

The single occupancy constraint is enforced on the average by the Lagrange multiplier λ in H_s . Note that the RVB order parameter Δ^s above, incorporates the link field A_{ij}^h , and is invariant under the following transformation: $A_{ij}^h \rightarrow A_{ij}^h + \theta_i - \theta_j$, and $b_{i\sigma} \rightarrow b_{i\sigma} e^{i\sigma\theta_i}$. Equations (14) and (15) define the effective Hamiltonian, H_{eff} . All the results presented in this paper are obtained using this Hamiltonian.

The Hamiltonian, H_J can be diagonalized by the Bogoliubov transformation,

$$b_{i\sigma} = \sum_m \left(u_{m\sigma}(i) \gamma_{m\sigma} - v_{m\sigma}(i) \gamma_{m-\sigma}^\dagger \right) . \quad (17)$$

The Bogoliubov factors, $u_{m\sigma}(i)$ and $v_{m\sigma}(i)$ can be reexpressed in terms of a single particle wave function $w_{m\sigma}(i)$ as $u(v)_{m\sigma}(i) = u(v)_m w_{m\sigma}(i)$, and $w_{m\sigma}$ is determined by the eigen equation

$$\xi_m w_{m\sigma}(i) = -\frac{J}{2} \sum_{j=nn(i)} \Delta_{ij}^s e^{i\sigma A_{ij}^h} w_{m\sigma}(j) . \quad (18)$$

The Hamiltonian, H_s is then diagonal in the γ_m representation, $H_s = \sum_{m\sigma} E_m \gamma_{m\sigma}^\dagger \gamma_{m\sigma}$, where $(E_m)_{\min} \sim \delta J$, as shown in [2]. The RVB order parameter is determined self consistently as

$$\Delta_{ij}^s = \sum_{m\sigma} e^{-i\sigma A_{ij}^h} w_{m\sigma}(i) w_{m\sigma}^*(j) (-u_m v_m) \left[1 + \sum_{\alpha} \langle \gamma_{m\alpha}^\dagger \gamma_{m\alpha} \rangle \right] . \quad (19)$$

In the following, we will always consider a real order parameter Δ_{ij}^s . In this case, it can be verified that $w_{m\sigma} = w_{m-\sigma}^*$ and that $(\Delta_{ij}^s)^* = \Delta_{ij}^s$. Finally, it is useful to note that

$$\langle \mathbf{S}_i \cdot \mathbf{S}_j \rangle = -\frac{1}{2} |\Delta^s|^2 < 0 \quad (20)$$

for nearest neighbors, ij . This is consistent with the notion that the b-RVB order parameter essentially characterizes the short range AF correlations.

III. GROUND STATE: SUPERCONDUCTING STATE WITH D-WAVE SYMMETRY

The b-RVB ground state is underpinned by the RVB order parameter, Δ_{ij}^s . At low temperatures, the holons are condensed. At $T = 0$, the ground state is superconducting. In this section, we will show that the superconducting order parameter has d -wave symmetry. This is due to the short range AF correlations. When a hole moves from a

lattice site $i + \hat{x}$ to $i + \hat{y}$, the string operator (4) picks up a minus sign on an average due to the short range AF correlations. We will show below that this translates into d -wave pairing of electrons.

The pair operator between nearest neighbor sites ij , $\hat{\Delta}_{ij}^{\text{SC}} \equiv \sum_{\sigma} \sigma c_{i\sigma} c_{j-\sigma}$, will now be reexpressed using the boson representation (2). For the purposes of this section, we find it convenient to define the quantities,

$$\Phi_i^s = \sum_{l \neq i} \theta_i(l) \sum_{\alpha} \alpha n_{l\alpha}^b ,$$

and

$$\Phi_i^0 = \sum_{l \neq i} \theta_i(l) .$$

Then, it is easy to verify that

$$\hat{\Delta}_{ij}^{\text{SC}} = e^{i\frac{1}{2}(\Phi_i^s + \Phi_j^s)} \hat{\Delta}_{ij}^0 , \quad (21)$$

where

$$\hat{\Delta}_{ij}^0 \equiv \left[(-1)^j (-1)^{N_h} e^{-i\Phi_j^0 - i\phi_{ij}^0} \right] h_i^{\dagger} h_j^{\dagger} \hat{\Delta}_{ij}^s \quad (22)$$

At low temperature, holons are Bose condensed, *viz.*, $\langle h_i^{\dagger} \rangle = h_0 e^{-i\phi_i^h}$, where ϕ_i^h is a fixed phase. Then, the superconducting order parameter can be written as a product of an amplitude and a phase as

$$\langle \hat{\Delta}_{ij}^{\text{SC}} \rangle = \langle \hat{\Delta}_{ij}^0 \rangle \langle e^{i\frac{1}{2}(\Phi_i^s + \Phi_j^s)} \rangle . \quad (23)$$

In the Appendix, we prove that the quantity $\langle \hat{\Delta}_{ij}^0 \rangle$ is a constant, independent of ij . The important physical quantity in this discussion is the phase factor, $e^{i\frac{1}{2}(\Phi_i^s + \Phi_j^s)}$ in (21).

First we note that a real superconducting order parameter will be established only when phase coherence is realized; *i.e.*, when spinons are paired up in the ground state such that $\langle e^{i\frac{1}{2}(\Phi_i^s + \Phi_j^s)} \rangle \neq 0$. Therefore, it should be clear that the superconducting transition temperature T_c is determined by phase coherence [8].

Besides the phase (de)coherence, the topological phase factor $e^{i\frac{1}{2}(\Phi_i^s + \Phi_j^s)}$ in (21) also determines the d -wave symmetry of the order parameter. To see this, let us compare $e^{i\frac{1}{2}(\Phi_i^s + \Phi_{i+\hat{x}}^s)}$ and $e^{i\frac{1}{2}(\Phi_i^s + \Phi_{i+\hat{y}}^s)}$. First, we rewrite

$$e^{i\frac{1}{2}(\Phi_i^s + \Phi_{i+\hat{\eta}}^s)} = e^{i(A_{ii+\hat{\eta}}^s + \Phi_{i+\hat{\eta}}^s)} ,$$

which holds true when two spinons at i and $i + \hat{\eta}$ are annihilated by $\hat{\Delta}_{ii+\hat{\eta}}^0$ in (21). We do so because $e^{iA_{ii+\hat{\eta}}^s}$ is well defined with $\langle A_{ii+\hat{\eta}}^s \rangle = 0$ at $T = 0$. Therefore, $\langle e^{iA_{ii+\hat{\eta}}^s} \rangle \approx e^{-\frac{1}{2}\langle (A_{ii+\hat{\eta}}^s)^2 \rangle} > 0$ in the ground state, and is independent of the nn link $ii + \hat{\eta}$. On the other hand, the difference between $e^{i\Phi_{i+\hat{x}}^s}$ and $e^{i\Phi_{i+\hat{y}}^s}$ leads to a phase change. Consider

$$e^{i\Phi_{i+\hat{x}}^s - i\Phi_{i+\hat{y}}^s} = e^{i[\theta_{i+\hat{x}}(i+\hat{y}) \sum_{\sigma} \sigma n_{i+\hat{y}\sigma}^b - \theta_{i+\hat{y}}(i+\hat{x}) \sum_{\sigma} \sigma n_{i+\hat{x}\sigma}^b] + iA_{i+\hat{x}, i+\hat{y}}^s} .$$

In the presence of short range AF correlations, one has $\sum_{\sigma} \sigma n_{i+\hat{y}\sigma}^b \approx \sum_{\sigma} \sigma n_{i+\hat{x}\sigma}^b$ such that

$$\begin{aligned} e^{i[\theta_{i+\hat{x}}(i+\hat{y}) \sum_{\sigma} \sigma n_{i+\hat{y}\sigma}^b - \theta_{i+\hat{y}}(i+\hat{x}) \sum_{\sigma} \sigma n_{i+\hat{x}\sigma}^b]} &\simeq e^{i[\theta_{i+\hat{x}}(i+\hat{y}) - \theta_{i+\hat{y}}(i+\hat{x})] \sum_{\sigma} \sigma n_{i+\hat{x}\sigma}^b} \\ &= -1 \end{aligned}$$

since $\theta_{i+\hat{x}}(i+\hat{y}) - \theta_{i+\hat{y}}(i+\hat{x}) = \pm\pi$. We emphasize that such a sign change is generally true on an average in an RVB background, and does *not* require long range AF order.

Thus, the presence of short range AF correlations among spins causes a d -wave like sign change between $\langle \hat{\Delta}_{ii+\hat{x}}^{\text{SC}} \rangle$ and $\langle \hat{\Delta}_{ii+\hat{y}}^{\text{SC}} \rangle$. If one only retains the nn pairing amplitude $\langle \hat{\Delta}_{ii+\hat{\eta}}^{\text{SC}} \rangle = \Delta^0 f_{i, i+\hat{\eta}}$, with $f_{i, i+\hat{x}}/f_{i, i+\hat{y}} = -1$, then in momentum space, one has the familiar d -wave form factor

$$\Delta_{\mathbf{k}}^{\text{SC}} = 2 \langle \hat{\Delta}_{ii+\hat{x}}^{\text{SC}} \rangle (\cos k_x a - \cos k_y a) \equiv \Delta_{\mathbf{k}}^0 \quad (24)$$

IV. ELEMENTARY EXCITATIONS

In this Section, we consider the confinement of bare spinon and holon excitations. Though the superconducting state at $T = 0$, $|\Psi_G\rangle$, is characterized by spinon (RVB) pairing and a holon condensate, it does not necessarily imply the presence of free spinon and holon excitations. In fact, we will show that creating a bare spinon or holon excitation out of the RVB vacuum costs infinite energy. On the other hand, we will show that neutral $S = 1$ excitations as well as quasiparticle excitations can be created from the ground state with finite energies.

A. Spin excitations

A single spinon excitation can be constructed as $\gamma_{m\sigma}^\dagger |\Psi_G\rangle$. As mentioned earlier, a mean field treatment of (13) yields the result, $E_m \sim \delta J$. Therefore, we may be tempted to conclude that the cost of creating a spinon excitation is finite. However, the presence of the long range topological gauge field A_{ij}^s , in the effective Hamiltonian for *holons* (15) suggests that the creation of a single spinon will be perceived as a half vortex, and consequently would be a high energy excitation. To see this, consider the energy cost of such an excitation, $\langle \Psi_G | \gamma_{m\sigma} H_h \gamma_{m\sigma}^\dagger | \Psi_G \rangle - E_0^h$, where $E_0^h = \langle \Psi_G | H_h | \Psi_G \rangle$. Now, consider $\langle \Psi_G | \gamma_{m\sigma} e^{iA_{ij}^f} h_i^\dagger h_j \gamma_{m\sigma} | \Psi_G \rangle$. From (17), we see that

$$\gamma_{m\sigma}^\dagger = \sum_l w_{m\sigma}(l) \left[u_m b_{l\sigma}^\dagger + v_m b_{l-\sigma} \right] .$$

Using this, we find

$$\langle \Psi_G | \gamma_{m\sigma} e^{iA_{ij}^f} h_i^\dagger h_j \gamma_{m\sigma} | \Psi_G \rangle = \sum_l \langle \Psi_G | e^{iA_{ij}^f} h_i^\dagger h_j | \Psi_G \rangle e^{i\frac{\sigma}{2}[\theta_i(l) - \theta_j(l)]} |w_{m\sigma}(l)|^2 .$$

Noting that $\theta_i(l) - \theta_j(l)$ is independent of l when summed over all links $\langle ij \rangle$ in H_h , and on using the normalization condition $\sum_l |w_{m\sigma}(l)|^2 = 1$, we get

$$\langle \Psi_G | \gamma_{m\sigma} H_h \gamma_{m\sigma}^\dagger | \Psi_G \rangle - E_0^h = E_0^h \times \left[\frac{1}{2N} \sum_{\langle ij \rangle} \cos \frac{1}{2} [\theta_i(l) - \theta_j(l)] - 1 \right] .$$

For a nearest neighbor link $\langle ij \rangle$ far away from any lattice site l , one has

$$|\theta_i(l) - \theta_j(l)| \sim \frac{a |\sin \alpha_{ij}|}{r_{li}}$$

where α_{ij} is the azimuthal angle of the link $\langle ij \rangle$ and $r_{li} \sim r_{lj}$ is the distance between l and $\langle ij \rangle$, and a is the lattice constant. Then, a summation over all nearest neighbor links $\langle ij \rangle$ leads to

$$\frac{1}{2N} \sum_{\langle ij \rangle} \cos \frac{1}{2} [\theta_i(l) - \theta_j(l)] - 1 \sim \ln \frac{L}{a} ,$$

L being the size of the sample. We thus see that it costs a logarithmically divergent energy to create a single spinon excitation. Consequently, single spinon excitations are not allowed in the superconducting ground state at $T = 0$. We reemphasize the crucial role played by the topological gauge field A_{ij}^s , leading to this result.

The above analysis suggests that a logarithmic divergence in the excitation energy can be avoided by pairing of spinons (binding of a vortex-antivortex pair). Such an excitation is nothing but a neutral $S = 1$ excitation. For example, consider a spin excitation created by $S_i^z, S_i^z | \Psi_G \rangle$. At $T = 0$, the relevant term in $S_i^z | \Psi_G \rangle$ is $\gamma_{m\sigma}^\dagger \gamma_{m'-\sigma}^\dagger | \Psi_G \rangle$. The excitation energy can be readily determined from the mean field theory of H_s , as

$$\langle \Psi_G | \gamma_{m'-\sigma} \gamma_{m\sigma} H_s \gamma_{m\sigma}^\dagger \gamma_{m'-\sigma}^\dagger | \Psi_G \rangle - E_0^s = E_m + E_{m'} ,$$

which is finite. Now let us check the vortex energy in the charge sector, H_h . A straightforward calculation yields

$$\begin{aligned} & \langle \Psi_G | \gamma_{m'-\sigma} \gamma_{m\sigma} H_h \gamma_{m\sigma}^\dagger \gamma_{m'-\sigma}^\dagger | \Psi_G \rangle - E_0^h \\ &= \frac{E_0^h}{2N} \sum_{\langle ij \rangle} \sum_{l_1 l_2} |w_{m\sigma}(l_1)|^2 |w_{m'-\sigma}(l_2)|^2 \cos \left\{ \frac{1}{2} [\theta_i(l_1) - \theta_j(l_1)] - \frac{1}{2} [\theta_i(l_2) - \theta_j(l_2)] \right\} - E_0^h . \end{aligned}$$

In obtaining the above, we have made repeated use of

$$h_i e^{\frac{i}{2}\Phi_j^h} = e^{\frac{i}{2}\theta_j(i)} e^{\frac{i}{2}\Phi_j^h} h_i ,$$

$$b_{i\sigma} e^{\frac{i}{2}\Phi_j^b} = e^{i\frac{\sigma}{2}\theta_j(i)} e^{i\frac{\sigma}{2}\Phi_j^b} b_{i\sigma} ,$$

etc. Note that the factor

$$\frac{1}{2}[\theta_i(l_1) - \theta_j(l_1)] - \frac{1}{2}[\theta_i(l_2) - \theta_j(l_2)]$$

vanishes when $l_1 = l_2$. It has a dipole configuration when $l_1 \neq l_2$, l_1 and l_2 being the cores of the spinon vortex-antivortex pair. The corresponding energy is no longer logarithmically divergent, since the factor $|w_{m\sigma}(l_1)|^2 |w_{m'-\sigma}(l_2)|^2$ restricts l_1 and l_2 within a characteristic length scale of the spinon wave packet, l_c . Hence, we get (in the continuum limit)

$$\begin{aligned} & \langle \Psi_G | \gamma_{m'-\sigma} \gamma_{m\sigma} H_h \gamma_{m\sigma}^\dagger \gamma_{m'-\sigma}^\dagger | \Psi_G \rangle - E_0^h \\ & \sim \frac{E_0^h}{N} \sum_{l_1 l_2} |w_{m\sigma}(l_1)|^2 |w_{m'-\sigma}(l_2)|^2 \ln \frac{|\mathbf{r}_{l_1} - \mathbf{r}_{l_2}|}{l_c} . \end{aligned}$$

Thus, an $S = 1$ spin excitation comprising a pair of spinons costs a finite energy, and is an elementary excitation. It does not decay into free spinons, due to a logarithmic confining force between the constituent spinons. The excitation energy has a lower bound, $E_g \gtrsim 2E_s$, where $E_s = (E_m)_{\min} \sim \delta J$.

To conclude this subsection, we note that an $S = 1$ excitation created by $S_i^+ = (-1)^i b_{i\uparrow}^\dagger b_{i\downarrow} e^{i\Phi_i^h}$ should also have a finite excitation energy, as required by spin rotational symmetry. This can be verified by an explicit calculation. In this case, the relevant excitation at $T = 0$ is of the form $\gamma_{m\uparrow}^\dagger \gamma_{m'\uparrow}^\dagger e^{i\Phi_i^h} | \Psi_G \rangle$. Following the same steps as above, we find that the logarithmic divergence is avoided by a cancellation between the 2π vortex Φ_i^h and two π vortices introduced by $\gamma_{m\uparrow}^\dagger \gamma_{m'\uparrow}^\dagger$. In the following subsection, we will discuss an analogous effect which leads to a finite energy for the quasiparticle excitation.

B. Quasiparticle excitations

A quasiparticle (or a quasihole) excitation is obtained as a linear combination of $c_{l\sigma} | \Phi_G \rangle$ and $c_{l-\sigma}^\dagger | \Phi_G \rangle$. We now show that the emergence and stability of such an excitation is due to the confinement of bare spinon and holon excitations at $T = 0$.

From (2), we see that a quasihole excitation has the form

$$c_{l\sigma} | \Psi_G \rangle \propto h_l^\dagger b_{l\sigma} e^{\frac{i}{2}[\Phi_l^s - \sigma\Phi_l^h]} | \Psi_G \rangle , \quad (25)$$

viz., it comprises the holon, spinon and the vortex fields. Let us examine these three components individually.

First, we have already seen that $b_{l\sigma} | \Psi_G \rangle$ would cost an infinite energy in H_h . Similarly, a single holon added to the ground state will also cost a logarithmically diverging energy in H_s . It is easily seen that

$$\frac{\langle \Psi_G | h_l H_s h_l^\dagger | \Psi_G \rangle}{\langle \Psi_G | h_l h_l^\dagger | \Psi_G \rangle} - E_0^s = E_0^s \times \left[\frac{1}{2N} \sum_{\langle ij \rangle} \cos \frac{1}{2}[\theta_i(l) - \theta_j(l)] - 1 \right] , \quad (26)$$

which diverges logarithmically with the lattice size. The third component in (25) is the vortex field. Consider

$$\langle \Psi_G | e^{+\frac{i}{2}\sigma\Phi_i^h} H_h e^{-\frac{i}{2}\sigma\Phi_i^h} | \Psi_G \rangle - E_0^h = E_0^h \times \left[\frac{1}{2N} \sum_{\langle ij \rangle} \cos \frac{1}{2}[\theta_i(l) - \theta_j(l)] - 1 \right] ,$$

which is also logarithmically divergent. Similarly,

$$\langle \Psi_G | e^{-\frac{i}{2}\Phi_i^s} H_s e^{\frac{i}{2}\Phi_i^s} | \Psi_G \rangle - E_0^s = E_0^s \times \left[\frac{1}{2N} \sum_{\langle ij \rangle} \cos \frac{1}{2} [\theta_i(l) - \theta_j(l)] - 1 \right],$$

which diverges too. Therefore, in the ground state of the b-RVB, the quasiparticle cannot decay into independent holon, spinon and vortex excitations as it costs infinite energy. On the other hand, a quasiparticle excitation as a composite excitation has a finite energy. In this case, a cancellation of the vortex effects ensues on binding the holon and spinon excitations to the vortex fields.

Consider H_s . Clearly, $b_{l\sigma}|\Phi_G\rangle$ costs a finite energy, and we saw earlier that both $h_l^\dagger|\Phi_G\rangle$ and $e^{\frac{i}{2}\Phi_l^s}|\Phi_G\rangle$ separately cost infinite energies in H_s . In contrast, consider the combination $h_l^\dagger e^{\frac{i}{2}\Phi_l^s}$ in (25). We will show below that the vortex effects cancel. Now,

$$H_s h_l^\dagger e^{\frac{i}{2}\Phi_l^s} |\Psi_G\rangle = h_l^\dagger \left[-J_s \sum_{\langle ij \rangle \sigma} \left(e^{i\sigma A_{ij}^h} \right) b_{i\sigma}^\dagger b_{j-\sigma}^\dagger e^{i\sigma \chi_{ij}} + h.c. \right] e^{\frac{i}{2}\Phi_l^s} |\Psi_G\rangle,$$

where the phase twist $\chi_{ij} = 1/2 [\theta_i(l) - \theta_j(l)]$ if $(ij) \neq l$ and $\chi_{ij} = 0$ if $i = l$ or $j = l$, is responsible for the logarithmic energy in (26). This is just the statement that the holon is perceived as a half vortex through the term A_{ij}^h . Such a vortex effect will be canceled by $e^{\frac{i}{2}\Phi_l^s}$, since

$$H_s h_l^\dagger e^{\frac{i}{2}\Phi_l^s} |\Psi_G\rangle = h_l^\dagger e^{\frac{i}{2}\Phi_l^s} \left[-J_s \sum_{\langle ij \rangle \sigma} \left(e^{i\sigma A_{ij}^h} \right) b_{i\sigma}^\dagger b_{j-\sigma}^\dagger + h.c. \right] |\Psi_G\rangle.$$

Equivalently, $[H_s, h_l^\dagger e^{\frac{i}{2}\Phi_l^s}] = 0$, such that

$$\langle \Phi_G | h_l e^{-\frac{i}{2}\Phi_l^s} H_s h_l^\dagger e^{\frac{i}{2}\Phi_l^s} | \Phi_G \rangle = \langle \Phi_G | h_l h_l^\dagger H_s | \Phi_G \rangle$$

is finite. Consequently, $c_{i\sigma}|\Psi_G\rangle$ defined in (25) must have a finite matrix element in H_s .

A similar analysis can be carried out for the matrix elements of H_h . In this case, $h_l^\dagger|\Psi_G\rangle$ and $e^{\frac{i}{2}\Phi_l^s}|\Psi_G\rangle$ cost finite energies in H_h , but $b_{l\sigma}|\Psi_G\rangle$ and $e^{-\frac{i}{2}\sigma\Phi_l^h}|\Psi_G\rangle$ yield divergent energies. The vortex effects can be cancelled by constructing the combination $b_{l\sigma} e^{-\frac{i}{2}\sigma\Phi_l^h}$, viz., $[H_h, b_{l\sigma} e^{-\frac{i}{2}\sigma\Phi_l^h}] = 0$, which shows that the vortex effect induced by $b_{l\sigma}$ in H_h is cancelled by the effect of $e^{-\frac{i}{2}\sigma\Phi_l^h}$.

Note that only the vortex phase factor $e^{\frac{i}{2}[\Phi_l^s - \sigma\Phi_l^h]}$ is single valued. Neither $e^{\frac{i}{2}\Phi_l^s}$ nor $e^{-\frac{i}{2}\sigma\Phi_l^h}$ are single valued by themselves [10]. This means that $h_l^\dagger e^{\frac{i}{2}\Phi_l^s}$ and $b_{l\sigma} e^{-\frac{i}{2}\sigma\Phi_l^h}$ cannot be independent physical objects. Therefore, the holon and spinon fields in (25) have to be bound to the vortex field $e^{\frac{i}{2}[\Phi_l^s - \sigma\Phi_l^h]}$ as a whole, such that the quasiparticle excitation is well defined and stable in the superconducting state.

Now that we have demonstrated the stability of the quasiparticle excitation, we determine its energy. Consider

$$\begin{aligned} & [H_s, c_{i\sigma}] \\ &= -J_s \sum_{m=nn(i)} \left[\sum_{\alpha} b_{i\alpha}^\dagger b_{m-\alpha}^\dagger e^{i\alpha A_{im}^h} + h.c. \right] c_{i\sigma} \\ &+ c_{i\sigma} J_s \sum_{m=nn(i)} \left(\sum_{\alpha} b_{i\alpha}^\dagger b_{m-\alpha}^\dagger e^{i\alpha A_{im}^h} + h.c. \right) \\ &= -J_s \sum_{m=nn(i)} h_i^\dagger e^{i\hat{\Theta}_{i\sigma}} \left(\sum_{\alpha} b_{i\alpha}^\dagger b_{m-\alpha}^\dagger e^{i\alpha A_{im}^h} e^{i\frac{\sigma}{2}\theta_i(m)} + h.c. \right) b_{i\sigma} \\ &+ J_s \sum_{m=nn(i)} h_i^\dagger e^{i\hat{\Theta}_{i\sigma}} b_{i\sigma} \left(\sum_{\alpha} b_{i\alpha}^\dagger b_{m-\alpha}^\dagger e^{i\alpha A_{im}^h} + h.c. \right) \end{aligned}$$

Each term in the above expression involves three spinon operators and can be linearized using the mean field approximation (16). Then,

$$[H_s, c_{i\sigma}] \simeq 10J_s\Delta^s c_{i\sigma} + J_s \sum_{m=nn(i)} h_i^\dagger e^{i\hat{\Theta}_{i\sigma}} b_{m-\sigma}^\dagger e^{i\sigma A_{im}^h} \langle b_{i\sigma} b_{i\sigma}^\dagger \rangle$$

(noting that terms involving $e^{i\frac{\sigma}{2}\theta_i(m)}$ vanish after averaging over all possible $\theta_i(m)$). The second term on the rhs of the above equation can be rewritten, using (2) and (21). We get

$$[H_s, c_{i\sigma}] \approx 10\Delta^s J_s c_{i\sigma} - \frac{J_s}{\Delta^s} \sum_{m=nn(i)} \langle b_{i\sigma} b_{i\sigma}^\dagger \rangle \left(\frac{\Delta_{im}^{SC}}{h_0^2} \right) \sigma c_{m-\sigma}^\dagger.$$

In obtaining the above expression, we have approximated h_i^\dagger by h_m , since the holons are condensed with an expectation value, $\langle h^\dagger \rangle = h_0$ (its phase ϕ^h can be always set as zero due to the gauge choice discussed in Appendix). The factor of σ in the second term of the rhs comes from commuting the holon field h_m with $e^{-i\hat{\Theta}_{m-\sigma}}$. We have used the definition of the superconducting order parameter,

$$\begin{aligned} \Delta_{im}^{SC} &= \sum_{\sigma} \langle h_i^\dagger h_j^\dagger e^{-i\sigma A_{ij}^h} b_{i\sigma} b_{j-\sigma} (-\sigma)^i (\sigma)^m (-1)^{N_h} e^{\frac{i}{2}(\Phi_i^b + \Phi_m^b)} \rangle \\ &\simeq h_0^2 \Delta^s \langle (-\sigma)^i (\sigma)^m (-1)^{N_h} e^{\frac{i}{2}(\Phi_i^b + \Phi_m^b)} \rangle. \end{aligned}$$

Then in momentum space,

$$[H_s, c_{\mathbf{k}\sigma}] \approx 10\Delta^s J_s c_{\mathbf{k}\sigma} - \Delta_{\mathbf{k}}^d \sigma c_{-\mathbf{k}-\sigma}^\dagger, \quad (27)$$

where $\Delta_{\mathbf{k}}^d = \frac{(3-\delta)J}{4} \left(\frac{\Delta_{\mathbf{k}}^0}{h_0^2} \right)$. Here, we have used $\langle b_{i\sigma} b_{i\sigma}^\dagger \rangle = 1 + \langle b_{i\sigma}^\dagger b_{i\sigma} \rangle = (3-\delta)/2$ and $J_s = \frac{\Delta^s}{2} J$.

On the other hand, the hopping term will generally give rise to a kinetic energy [11]

$$[H_h, c_{\mathbf{k}\sigma}] = -(\epsilon_{\mathbf{k}} - \mu) c_{\mathbf{k}\sigma} \quad (28)$$

with $\epsilon_{\mathbf{k}} \sim t_h(\cos k_x + \cos k_y)$. Clearly, a quasiparticle operator can be constructed in the usual manner as

$$\alpha_{\mathbf{k}\sigma}^\dagger = \tilde{u}_{\mathbf{k}} c_{\mathbf{k}\sigma}^\dagger - \sigma \tilde{v}_{\mathbf{k}} c_{-\mathbf{k}-\sigma}$$

where $\tilde{u}_{\mathbf{k}}^2 + \tilde{v}_{\mathbf{k}}^2 = 1$. On imposing the condition $[H_s, \alpha_{\mathbf{k}\sigma}^\dagger] = E_{\mathbf{k}}^{qp} \alpha_{\mathbf{k}\sigma}^\dagger$ and using equations (27) and (28), it is straightforward to get the standard expressions, $E_{\mathbf{k}}^{qp} = \sqrt{(\epsilon_{\mathbf{k}} - \mu)^2 + (\Delta_{\mathbf{k}}^d)^2}$, $\tilde{u}_{\mathbf{k}} = \frac{1}{\sqrt{2}} \left(1 + \frac{\epsilon_{\mathbf{k}} - \mu}{E_{\mathbf{k}}^{qp}} \right)^{1/2}$, and $\tilde{v}_{\mathbf{k}} = \frac{1}{\sqrt{2}} \left(1 - \frac{\epsilon_{\mathbf{k}} - \mu}{E_{\mathbf{k}}^{qp}} \right)^{1/2} \text{sgn}(\Delta_{\mathbf{k}}^d)$. The first term on the rhs of (27) can be absorbed in the definition of the chemical potential, μ . The latter can be determined within mean field theory in the usual manner, by fixing the particle number.

The d-wave gap $\Delta_{\mathbf{k}}^d$ vanishes along the two nodal lines, $k_x = \pm k_y$. Thus, the quasiparticle spectrum has four nodes at $\epsilon_{k_f} - \mu = 0$.

It is important to note that even though the confining force between the holon, spinon, and vortex constituents within a quasiparticle is weak (logarithmic), the quasiparticle should be a well defined, long lived excitation. Even if we neglect the logarithmic energy cost that keeps them bound (which becomes important when the components are separated far away), the mean field energies for holon and spinon constituents are zero and E_m , respectively. Thus, for $E_{\mathbf{k}}^{qp} < (E_m)_{\min} \equiv E_s$, the quasiparticle energy is always lower than its constituents. This implies that the nodal quasiparticle excitation will be observed as a sharp spectral feature. Note that the vortex field in (25) also provides a confining potential that glues the spin and charge degrees of freedom. This lowers the quasiparticle energy, in comparison to the mean field energies of the spinon and holon excitations.

However, at higher energies, the signature of the composite quasiparticle will be seen in the spectral function. For energies $\omega > E_s$, the spectral function, $A(\mathbf{k}, \omega) = -\frac{1}{\pi} \text{Im} G(\mathbf{k}, \omega) \text{sgn}(\omega)$, displays a composite structure in the form of a broad peak and spectral weight at higher energies [12,10]. Therefore, even though low energy quasiparticles in the b-RVB superconductor seemingly look like their counterparts in a conventional d-wave BCS superconductor, they are easily distinguished from the latter at higher energies, by a broad incoherent structure. This reflects the composite nature of the RVB vacuum (*i.e.*, spin charge separation).

It is clear that the width of the quasiparticle would increase if $E_{\mathbf{k}}^{qp}$ becomes comparable to, or greater than, E_s . Typically, this would first happen around the corners of the Brillouin zone, $(\pm\pi, 0)$ and $(0, \pm\pi)$. In these regions, $\Delta_{\mathbf{k}}^d$

and $E_{\mathbf{k}}^{qp}$ attain their maximal values. Since the mean field excitation energies of the holon and spinon constituents are now comparable to, or smaller than, $E_{\mathbf{k}}^{qp}$, the quasiparticle will be seen as a loosely bound spinon-holon composite. This is consistent with results from exact diagonalization [13,14].

A similar effect also occurs as the hole concentration is reduced. As $\delta \rightarrow 0$, $\Delta_{\mathbf{k}}^d$ is finite (recall the factor of h_0^2 in the denominator of $\Delta_{\mathbf{k}}^d$), whereas $E_s \sim J\delta \rightarrow 0$. Therefore, at small δ , the sharp quasiparticle peak evolves into a broad feature quickly as one moves away from the nodal regions to the corners of the Brillouin zone. Eventually, for low enough doping, the spectral function becomes very broad away from the nodal points ($\pm\pi/2, \pm\pi/2$), and is best described in terms of spin charge separation rather than a bound excitation. Elsewhere, we discussed this in detail for the case of a single hole doped into the antiferromagnetic Mott insulator [15].

V. SUMMARY

In this paper, we discussed the ground state and the nature of excitations in a bosonic resonating valence bond (b-RVB) superconductor. We considered an effective Hamiltonian for the spin and charge degrees of freedom. The effective Hamiltonian was derived by rewriting the $t - J$ model using the phase string decomposition. This decomposition can be thought of as a bosonization scheme for the $t - J$ model. The ground state at $T = 0$ is a d-wave superconducting state which arises from RVB pairing and holon condensation.

The physical reason behind d -wave pairing has a simple explanation. Suppose one of the holes forming the Cooper pair is located at site i . As the other hole belonging to the pair is transported from site $i + \hat{x}$ to $i + \hat{y}$, say, via $i + \hat{x} + \hat{y}$ (the shortest path), it picks up a phase string, $\sigma_{i+\hat{x}+\hat{y}}\sigma_{i+\hat{y}}$, due to an irreparable displacement of the Marshall sign that occurs due to hole motion [10]. Owing to the short range AF correlations, $\sigma_{i+\hat{x}+\hat{y}}\sigma_{i+\hat{y}} < 0$ on the average, which implies that the Cooper pair at sites i and $i + \hat{y}$ changes sign relative to the pair at i and $i + \hat{x}$.

The superconducting condensation in the b-RVB and conventional d-RVB theories [9] both involve the RVB pairing of spinons and holon condensation. The main distinction between these two classes of theories lies in the origins of kinetic energy frustration. In the b-RVB theory, the kinetic energy of holons is strongly frustrated, due to the short range AF correlations that are carefully incorporated. This kinetic energy frustration is realized as the phase string effect. A partial recovery of the frustrated kinetic energy occurs at the superconducting transition. However, spinon and holon excitations out of the b-RVB vacuum still exhibit the signature of strong mutual frustration between themselves: a single spinon or holon excitation always invites a severe response from the other degrees of freedom, causing a logarithmically divergent energy. This is the origin of the low energy quasiparticle, by spin charge recombination.

We showed that in the superconducting state, $S = 1$ excitations composed of spinon pairs have finite energies, although single spinons would cost a logarithmic energy. Similarly, a composite quasiparticle excitation, carrying both spin and charge, is well defined and has a finite energy. Though the quasiparticle operator we constructed has a superficial similarity with the nodal quasiparticle in a conventional d -wave BCS superconductor, there are important differences. The first is that neutral $S = 1$ excitations in the b-RVB state, to leading order, are independent of quasiparticle excitations. They occur as bound spinon pairs, whereas in BCS theory, the $S = 1$ excitation is built out of quasiparticles. A second difference is that the composite character of the quasiparticle can be seen in the spectral function, at high energies. For low doping, even the low energy quasiparticle is unstable at the Brillouin zone corners. It should be noted that the quasiparticle in the b-RVB theory becomes incoherent at T_c . As discussed in Ref. [8], the superconducting transition occurs when spinons are deconfined such that free spinon-vortices destroy the phase coherence of the order parameter (23). The proliferation of spinon vortices leads to the screening of the logarithmic confining potential, and the quasiparticle decays into its constituents. These differences in the excitation spectra reflect the fundamental difference between a BCS superconductor and an RVB superconductor. The former is constructed from a Fermi sea of electrons, whereas the latter is built from an RVB vacuum which describes a doped antiferromagnetic Mott insulator.

Though spin charge separation is the key feature in the description of the doped Mott insulator, an important point we discussed in this paper is that bare holons and spinons need not necessarily exist as excitations of the RVB vacuum. The fact that a single holon or spinon cannot be created or annihilated in the system at $T = 0$ is consistent with the requirement of the no double occupancy constraint at the global level. In contrast, an $S = 1$ spin excitation or a quasiparticle excitation does not violate such a constraint. The confinement of single spinon and holon excitations, along with the emergence of $S = 1$ spin excitations and quasiparticles as the low lying elementary excitations in the b-RVB theory has furthered our understanding of the concepts of the RVB state and spin charge separation. In fact, deconfinement of spinons and holons plays a key role in determining the superconducting transition and in the description of the high temperature phases of the b-RVB theory. The phase string decomposition (2) provides a natural realization of a two dimensional fermion-boson transmutation envisaged by Baskaran, during the early years

of the RVB theory [16]. Recently, Sarker has discussed the recombination of spin and charge degrees of freedom in the superconducting state of the $t-J$ model, using a Schwinger boson representation [17]. Spin-charge recombination in the superconducting phase was also conjectured previously by Wen, Lee and others [18] within the slave-boson framework. In contrast, fractionalization of electrons or spin charge separation of the elementary excitations persists in the superconducting state obtained within Z_2 gauge theory [19].

Finally, in this paper, we were mainly interested in the stability and finite energy of the nodal quasiparticle. We showed that the composite character of this excitation can be observed clearly as a function of doping. As the doping is reduced, the quasiparticle peak around the corners of the Brillouin zone would broaden and the spectral function is better described in terms of spin charge separation. A quantitative analysis of this phenomenon in terms of the life time of the quasiparticle excitation, and a comparison with experimental results available on the evolution of the Fermi surface with doping, will be discussed elsewhere.

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APPENDIX A: APPENDIX: EVALUATION OF $\langle \hat{\Delta}_{IJ}^0 \rangle$

The amplitude of the superconducting order parameter in (23) can be expressed by

$$\langle \hat{\Delta}_{ij}^0 \rangle = g_{ij} h_0^2 \Delta^s \quad (\text{A1})$$

with

$$g_{ij} \equiv (-1)^j (-1)^{N_h} e^{-i(\Phi_j^0 + 2\phi_j^h) - i[\phi_{ij}^0 + (\phi_i^h - \phi_j^h)]} \quad (\text{A2})$$

The link quantity g_{ij} defined in (A2) may be rewritten as:

$$g_{ij} = (-1)^j (-1)^{N_h} e^{-i(\Phi_j^0 + 2\phi_j^h)} e^{-i\bar{\phi}_{ij}^0} \quad (\text{A3})$$

with $\bar{\phi}_{ij}^0 \equiv \phi_{ij}^0 + (\phi_i^h - \phi_j^h)$.

By using $\theta_i(j) - \theta_j(i) = \pm\pi$, one can generally show that

$$g_{ij} = g_0 e^{-i \sum_c 2\bar{\phi}_{lm}^0} e^{-i\bar{\phi}_{ij}^0}$$

where $g_0 \equiv (-1)^{j_0} (-1)^{N_h} e^{-i(\Phi_{j_0}^0 + 2\phi_{j_0}^h)}$ and c represents an arbitrary path composed of nn links on the lattice which connects the site j and a reference site j_0 . Note that, as $\sum_{\square} 2\bar{\phi}_{ij}^0 = \sum_{\square} 2\phi_{ij}^0 = \pm 2\pi$, g_{ij} is independent of the choice of the path c .

Once j_0 is fixed, g_{ij} on every link will be determined. For example, on the horizontal bonds connected to j_0 : $(j_0, j_0 \pm \hat{x})$, $(j_0, j_0 \pm 2\hat{x})$, $(j_0, j_0 \pm 3\hat{x})$, ...

$$\begin{aligned} g_{j_0, j_0 \pm \hat{x}} &= g_0 e^{-i2\bar{\phi}_{j_0 \pm \hat{x}, j_0}^0 - i\bar{\phi}_{j_0, j_0 \pm \hat{x}}^0} = g_0 e^{i\bar{\phi}_{j_0, j_0 \pm \hat{x}}^0} \\ g_{j_0 \pm \hat{x}, j_0 \pm 2\hat{x}} &= g_0 e^{-i(2\bar{\phi}_{j_0 \pm 2\hat{x}, j_0 \pm \hat{x}}^0 + 2\bar{\phi}_{j_0 \pm \hat{x}, j_0}^0) - i\bar{\phi}_{j_0 \pm \hat{x}, j_0 \pm 2\hat{x}}^0} \\ &\dots \end{aligned} \quad (\text{A4})$$

Similarly on the vertical bonds connected to j_0 : $(j_0, j_0 \pm \hat{y})$, $(j_0, j_0 \pm 2\hat{y})$, $(j_0, j_0 \pm 3\hat{y})$, ...

$$\begin{aligned} g_{j_0, j_0 \pm \hat{y}} &= g_0 e^{-i2\bar{\phi}_{j_0 \pm \hat{y}, j_0}^0 - i\bar{\phi}_{j_0, j_0 \pm \hat{y}}^0} = g_0 e^{i\bar{\phi}_{j_0, j_0 \pm \hat{y}}^0} \\ g_{j_0 \pm \hat{y}, j_0 \pm 2\hat{y}} &= g_0 e^{-i(2\bar{\phi}_{j_0 \pm 2\hat{y}, j_0 \pm \hat{y}}^0 + 2\bar{\phi}_{j_0 \pm \hat{y}, j_0}^0) - i\bar{\phi}_{j_0 \pm \hat{y}, j_0 \pm 2\hat{y}}^0} \\ &\dots \end{aligned} \quad (\text{A5})$$

Now let us consider the choice of the gauge in $\vec{\phi}_{ij}^0$ which satisfies

$$\sum_{\square} \vec{\phi}_{ij}^0 = \pm \pi \quad (\text{A6})$$

The holon Hamiltonian in the holon condensed phase reduces to

$$H_h = -t_h h_0^2 \sum_{\langle ij \rangle} e^{i[A_{ij}^s - \vec{\phi}_{ij}^0]} + h.c. \quad (\text{A7})$$

Note that in the ground state all spinons are paired up such that A_{ij}^s is cancelled out and $\langle e^{iA_{ij}^s} \rangle \approx e^{-\frac{1}{2} \langle (A_{ij}^s)^2 \rangle}$ which is generally independent of (ij) . Then

$$\langle H_h \rangle = -t_h h_0^2 \langle e^{iA_{ij}^s} \rangle \sum_{\langle ij \rangle} e^{-i\vec{\phi}_{ij}^0} + h.c. \quad (\text{A8})$$

which is optimized at the staggered- π -flux gauge choice of

$$\begin{aligned} \vec{\phi}_{ii+\hat{x}}^0 &= \pm (-1)^i \pi/4 \\ \vec{\phi}_{ii+\hat{y}}^0 &= \mp (-1)^i \pi/4 \end{aligned} \quad (\text{A9})$$

satisfying (A6). In (A9), there are two choices of signs, \pm , for the staggered π flux at a given plaquette.

On averaging over these two equivalent gauge choices, we get

$$\begin{aligned} g_{j_0, j_0 \pm \hat{x}} &= g_0 \langle e^{i\vec{\phi}_{j_0, j_0 \pm \hat{x}}^0} \rangle = g_0 / \sqrt{2} \\ g_{j_0 \pm \hat{x}, j_0 \pm 2\hat{x}} &= g_0 \langle e^{-i\vec{\phi}_{j_0 \pm \hat{x}, j_0 \pm 2\hat{x}}^0} \rangle = g_0 / \sqrt{2} \\ &\dots \end{aligned}$$

In general

$$g_{ij} = g_0 / \sqrt{2} \quad (\text{A10})$$

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